INTRINSIC ULTRA CONTRACTIVITY OF SCHRÖDINGER SEMIGROUPS WITH POLYNOMIALLY GROWING POTENTIALS

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Abstract. The aim of this project is to prove that the heat kernel $k$ associated to the Schrödinger operator $A = -\sum_{j,k=1}^{n} \partial_k (a_{jk} \partial_j) + |x|^\alpha$, $\alpha > 2$, satisfies the estimate

$$k(t, x, y) \leq C e^{ct-b} \left( \frac{e^{-\frac{2\theta}{\alpha+2} |x|^{\alpha+\frac{2}{\alpha}}} \left( \frac{e^{-\frac{2\theta}{\alpha+2} |y|^{\alpha+\frac{2}{\alpha}}} \right)}{\frac{\alpha+2}{\alpha+1} |x|^{\alpha+1}} \right)$$

for large $x, y \in \mathbb{R}^n$, $0 < t \leq 1$, $0 < \theta \leq 1$ and $b \geq \frac{\alpha+2}{\alpha+1}$.

In the first part we propose to prove such estimates for the heat kernel $p$ associated to the operator $B = -\Delta + |x|^\alpha$. Here one can choose between the approach due to Davies and Simon (see [1, Section 4.5]) or the approach using Lyapunov functions (see [2, Sections 5 and 6]).

The last part is dedicated to the estimates (0.1). This can be obtained by domination arguments, using the result of the first part and a log-Sobolev inequality (see [3]). As an application one obtains estimates of the eigenfunctions of $A$.

References


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