Many time-dependent partial differential equations can be expressed as an abstract Cauchy problem
\[ u'(t) = (A + B)u, \quad t \geq 0, \quad u(0) = u_0 \] (1)
where the generator of the semigroup is the sum of two operators \( A \) and \( B \). Often the solutions of the sub-problems
\[ v'(t) = Av, \quad v(0) = v_0 \]
and
\[ w'(t) = Bw, \quad w(0) = w_0 \]
are explicitly known or can be approximated much more efficiently than the solution \( u(t) \) of the full problem (1). In this situation, the Lie-Trotter splitting
\[ u(n\tau) = e^{n\tau(A+B)}u_0 \approx (e^{\tau A}e^{\tau B})^n u_0 \] (2)
or the Strang splitting
\[ u(n\tau) = e^{n\tau(A+B)}u_0 \approx (e^{\tau B/2}e^{\tau A}e^{\tau B/2})^n u_0 \] (3)
can be applied to approximate the solution of (1) at discrete times \( n\tau \) (\( n \in \mathbb{N} \)), where \( \tau > 0 \) is the step-size and \( e^{tL} \) denotes the semigroup generated by the operator \( L \in \{A, B, A+B\} \).

The goals of this project are

- to prove error bounds for the approximations (2) and (3),
- to show that the assumptions made in the proof allow an application of the methods to the linear Schrödinger equation, and
- to confirm the predicted error behavior by numerical experiments.

Participants who are interested in the third part of the project must be familiar with MATLAB.

Reference:

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